

DC Circuits

Key Info:

Voltage - An electric potential difference across two points, calculated as the electromotive force minus current times resistance.

Current - The rate at which charge flows, denoted in amperes. A voltage potential V creates an electric field, which motivates charge to flow.

Resistivity - The resistance to the flow of charge. For a wire, resistivity is calculated with length, cross-sectional area, and a material-specific resistivity constant ρ . Resistors in series sum regularly, resistors in series inversely sum.

Ohm's Law - Voltage = Current * Resistance. This makes sense, as if we hold the voltage constant, a higher current means a lower resistance, and a lower current means a higher resistance.

Node - A point where two or more elements of a circuit are connected.

Branch - A part of a circuit with only one element (typically wire), with nodes at each end.

The Junction Rule: The total current entering a node must equal the total current out of the node.

The Loop Rule: Across a closed loop, the sum of all potential changes will be zero.

Key Equations:

$$V = \mathcal{E} - iR$$

$$\Delta U = q\Delta V$$

$$P = i^2R = \frac{V^2}{R}$$

$$i = \frac{dq}{dt}$$

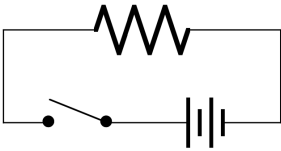
$$R = \frac{\rho L}{A}$$

$$R_{series} = R_1 + R_2 \dots$$

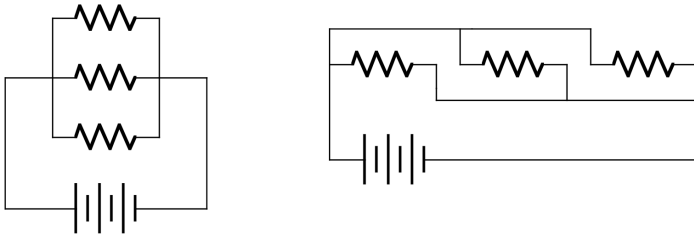
$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} \dots$$

Circuitry:

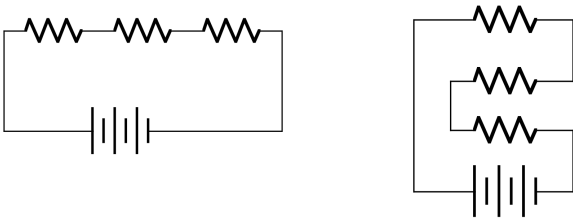
A default circuit with a switch, voltage source, and resistance:



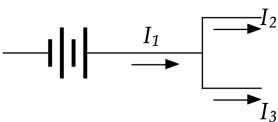
A circuit with resistors in parallel, which sum inversely:



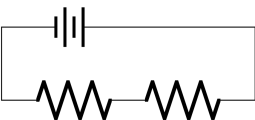
A circuit with resistors in series, which sum directly:



A circuit representing Kirchhoff's first law, where $i_1 = i_2 + i_3$

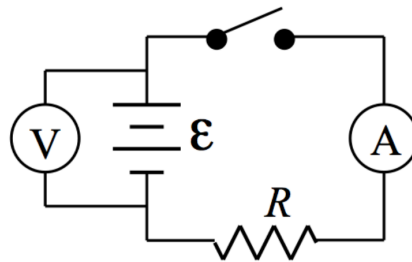


A circuit representing Kirchhoff's second law, where the voltage difference when traversing the entire loop from one point back to itself will equal zero. In other words, the potential increase due to V_0 will be equal to the potential decrease due to iR_1 and iR_2 .

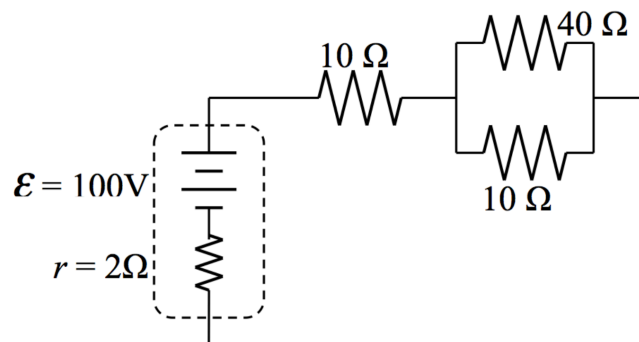


Practice Questions

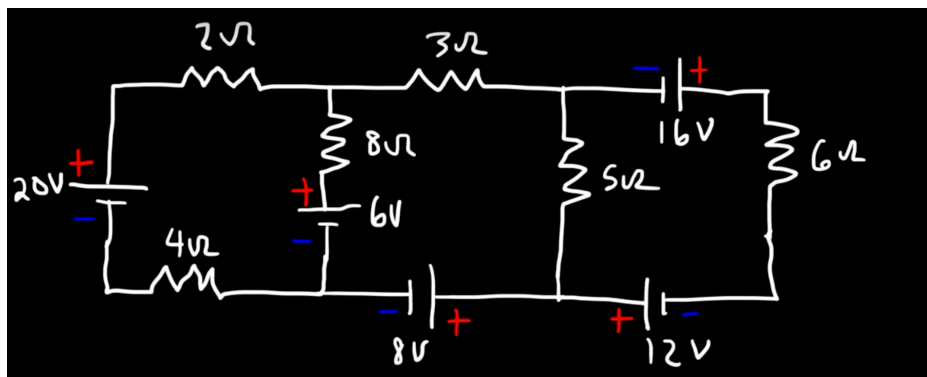
1. In the circuit below, a battery with EMF ϵ is connected to a switch, a resistor with resistance R , a voltmeter V , and an ammeter A . The voltmeter measures a potential of 9 V across the battery before the switch is closed. After the switch is closed, the potential goes down to 8.4 , and the ammeter reads 0.8 amps. What is the resistance R ?



2. A circuit with two $10\ \Omega$ resistors and a $40\ \Omega$ resistor is connected to a 100 V emf with an internal resistance of $2\ \Omega$. How much power is dissipated due to the $40\ \Omega$ resistor?



3. Massive doozy incoming. Find the power dissipated by the $8\ \Omega$ resistor in the circuit below.



Solutions:

1.

$$V = \varepsilon - iR \quad R = \frac{V - \varepsilon}{i} = \frac{(8.4 - 9)}{0.8} = 0.75\Omega \text{ * Manipulate voltage equation}$$

2.

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{10} + \frac{1}{40} \quad R_{\text{parallel}} = 8\Omega \text{ *Find the equivalent resistor of parallel combination}$$

$$R_{\text{series}} = R_{\text{parallel}} + R_{10} + r = 8 + 10 + 2 = 20\Omega \text{ *Find the equivalent resistor of the total series combination}$$

$$i = \frac{V}{R} = \frac{100}{20} = 5A \text{ *However, only 1/5th (40 / 10+40) of total current flows through the 40 ohm resistor.}$$

$$P = i^2 R = 1^2 \cdot 40 = 40W \text{ *Plug into the power relationship}$$

3:

3.

Label currents and loops as so.

Set up loop equations:

$$L_1: 20 - 2i_1 - 8i_2 - 6 - 4i_1 = 0 \rightarrow 14 = 6i_1 + 8i_2 \rightarrow 7 = 3i_1 + 4i_2$$

$$L_2: 6 + 8i_2 - 3(i_1 - i_2) - 5i_3 - 8 = 0 \rightarrow 2 = -3i_1 + 11i_2 - 5i_3$$

$$L_3: 5i_3 + 16 - 6(i_1 - i_2 - i_3) + 12 = 0 \rightarrow 28 = 6i_1 - 6i_2 - 11i_3$$

Combine / Solve system of equations:

$$\begin{aligned} 3i_1 + 4i_2 &= 7 \\ -3i_1 + 11i_2 - 5i_3 &= 2 \\ 6i_1 - 6i_2 - 11i_3 &= 28 \end{aligned}$$

Combine equations 2 and 3 to cancel i_3 . To do so we need to multiply equation 2 by -1 and the 3rd by 5

$$\begin{aligned} -11(-3i_1 + 11i_2 - 5i_3 = 2) \\ + 5(6i_1 - 6i_2 - 11i_3 = 28) \end{aligned}$$

$$\begin{aligned} 63i_1 - 151i_2 &= 118 \\ + 2(3i_1 + 4i_2 = 7) \end{aligned}$$

$$-235i_2 = -24 \rightarrow \boxed{I_2 = 0.1234 A}$$

Then multiply equation 1 by 21

$$P = i^2 R = (0.1234)^2 \cdot 8 = \boxed{0.122 W}$$